Here are some problems that might be useful to go through if you haven't yet.

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1 What gates can be generated with H and S?

2 Verify that

$$\begin{aligned}
&XY = iZ = -YX, \\
&YZ = iX = -ZY, \\
&ZX = iY = -XZ, \\
&HX = ZH \Rightarrow HXH = Z \; and \; HZH = X, \\
&SX = YS \Rightarrow SXS^{\dagger} = Y \; and \; S^{\dagger}YS = X \\
&S^{\dagger}X = -YS^{\dagger} \Rightarrow \; S^{\dagger}XS = -Y \; and \; SYS^{\dagger} = -X
\end{aligned}$$

3 C_{ij} is the Cnot gate acting on qubits i and j with i being the control and j being the target. $C_{12} \mid n \; m\rangle = \mid n \; (m \oplus n)\rangle$ where $n, m \in \{0, 1\}$. Verify that $C_{ij} X_i C_{ij}^{\dagger} = X_i X_j$ and commutes with every other pauli $X_{n \neq i}$ operator where X_i means pauli X acting on the ith qubit. Also verify that $C_{ij} Z_j C_{ij}^{\dagger} = Z_i Z_j$.

4 The pauli group on n qubits G_n is the group generated by $< \{i^m\}, \{X_j\}, \{Y_j\}, \{Z_j\} >$ $, m \in \{0, 1, 2, 3\} \;\; j \in \{0, 1, ..., n\}$. e.g. For $g \in G_3, \; g = iX_1Z_3 = iX \otimes I \otimes Z$. When do two two elements in the pauli group commute?

5 When can a subgroup of the pauli group have simultaneous eigenvalues on a quantum state?